Measurement of Surface-Wave Statistics

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The motion of the ocean's surface is a random process describable in a statistical sense. In recent years there has been a significant improvement in the availability of instrumentation to measure the surface parameters, particularly in the short-gravity-wave and capillary-wave regimes. This paper will review the state-of-the-art of surface-wave measurement techniques, emphasizing the advantages and drawbacks of each technique.

I. Introduction

RECENTLY there has been increasing interest in measuring the parameters of the shorter gravity waves and the capillary waves. (Capillary waves are characterized by wavelengths, usually taken to be less than 1.7 cm, where the surface tension is an important factor in determining wave propagation.) This interest is an outgrowth of a desire to understand the wind wave interactions, the heat transfer from the ocean surface, and the interaction of electromagnetic (and acoustic) waves with the ocean surface.

The measurement of longer-wave components has been accomplished using accelerometers, pressure sensors, and wave staffs. The desire to measure the short-wavelength components has led to the development of new techniques described in following sections.

When discussing the desirability of measuring the parameters of surface waves, the quantities usually desired are one or all of the following: the spatial power-spectral-density (PSD) function of wave-height fluctuations or its Fourier transform, the spatial correlation coefficient, the probability density function of wave height, rms wave height, and rms wave slope. Of course, an accurate measure of the spatial PSD function or correlation coefficient of wave-height fluctuations permits the determination of rms wave height or rms wave slope.

The techniques developed for measuring the short gravity-wave and capillary-wave are: 1) wave-staff arrays; 2) optical reflection: a) slope-probability sensors, b) curvature-probability sensors, c) spatial PSD sensors; 3) optical refraction; 4) laser profilometers; 5) radar systems. These techniques are reviewed in the following sections.

II. Techniques of Measuring Surface Parameters

A. Wave-Staff Arrays

Wave staffs have been used for some time for the measurement of wave height. These staffs are of two types: the capacitance type and the resistance type (see Fig. 1). In both, the sea water is effectively an electrical short circuit. In one type, the capacitance between a dielectrically clad wire and the water is measured as the surface height changes. In the other type, the resistance of bare wire (usually nichrome) is modulated as the ocean height fluctuates. The capacitance-type wave staff has been found unsatisfactory at higher frequencies (and therefore shorter wavelengths) because of probe wetting.[1] Resistance-type wave staffs have been used at SRI[2,3] and elsewhere[1] to measure the temporal fluctuations of wave height up to 25 Hz.

The wave staff is a point measure of wave-height fluctuations, whose spatial resolution is probably limited by the meniscus around the probe. Although the wavelength limits have not yet been adequately defined, spatial wavelengths shorter than 1 cm can be resolved. To determine the spatial properties of the statistics of the ocean surface using a point sensor, it is required to have a dispersion relation that can relate wave length to frequency; otherwise, an array of wave staffs must be constructed to afford a direct measure of the spatial correlation coefficient. The dispersion relation, frequently quoted for free waves on the ocean's surface, has been found inappropriate for the wind-driven sea because the short gravity waves are often parasitic to longer wavelengths.

As a consequence, an array of wave staffs must be deployed to measure the spatial statistics. However, once the array is deployed, it affords a maximum measure of information about the surface and affords 24-hour-a-day operation. The relevant wave-height statistics, including the correlation coefficient, wave direction, and wave-packet lift-time are readily determined from an array of wave staffs. In Fig. 2, the one-dimensional spatial spectrum inferred from a wave-staff array will be shown and compared to that measured by an optical reflection technique. The limitations of wave-staff systems are associated with the requirement that the sensor be in contact with the water.

B. Optical Reflection

Within this classification of surface-wave sensors, a number of techniques have been developed; these will be considered following.

1. Slope-Probability Sensors

Slope probability density functions (PDF) can be inferred from the distribution of specular glints from the surface of the ocean when illuminated by nearly collimated light such as that from the sun, moon, or a laser. While this basic concept was used by Spooner,[4] over one hundred years ago (see also Hulbert,[5] Shuleikin,[6] Cox and Munk,[7] and Schooley,[8]) it is commonly referred to as the Cox and Munk technique because it was they who provided the theoretical and experimental foundation needed to establish it as a practical tool. This technique is based on the assumption that the scattering of electromagnetic radiation at optical frequencies from the surface of the ocean can be treated as a classical ray-optics problem. Recently, Krishnan and Peppers,[9] using scalar diffraction theory based on the Huygens-Fresnel principle, verified this assumption for radiation at optical frequencies and indicated the conditions for which higher-order diffraction corrections must be utilized.

The technique is best understood through reference to Fig. 3. From this geometry it is possible to write the condition on the wave slope $\tan \mu = |\nabla w|$ for receiving specularly reflected

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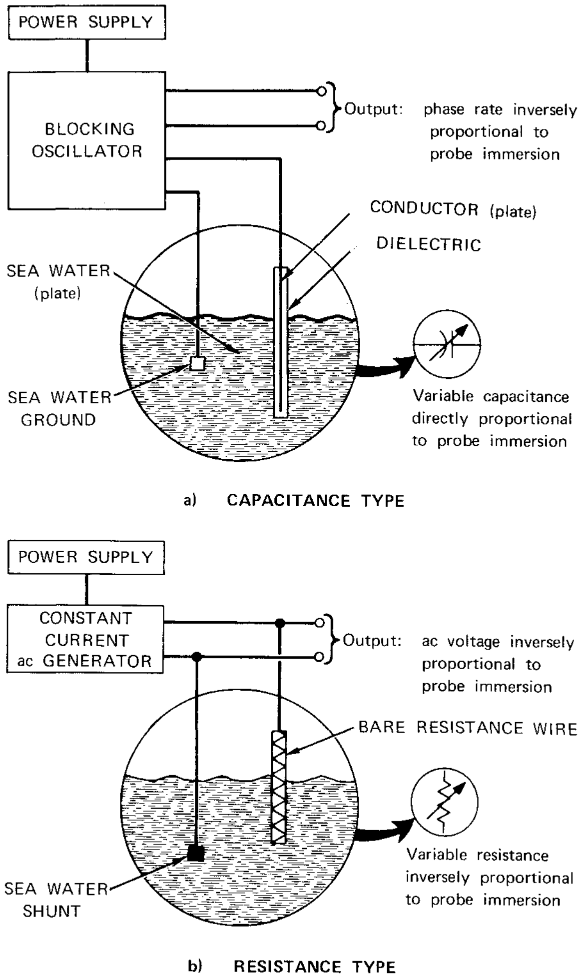

Fig. 1 Capacitance and resistance wave-measuring devices.

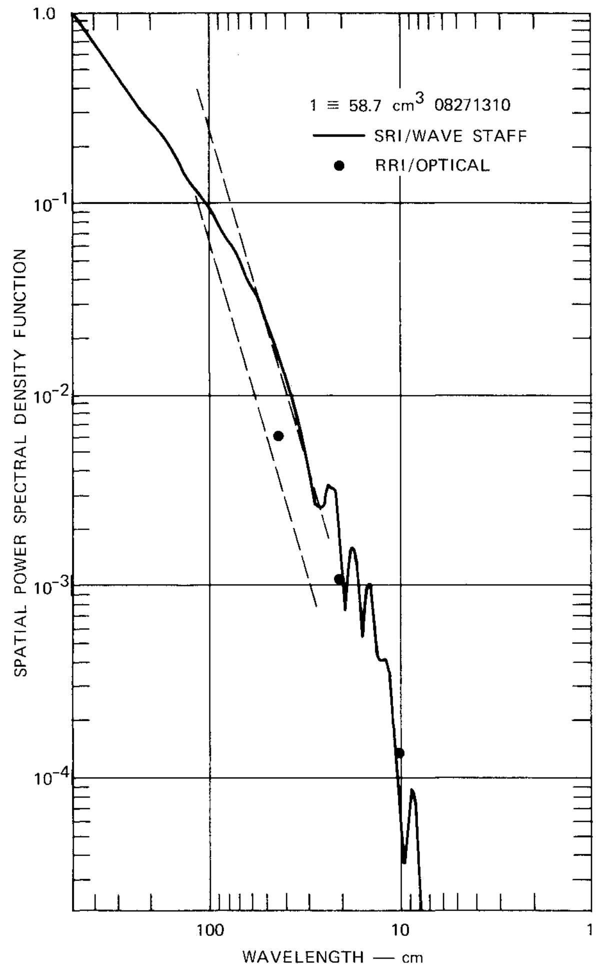

Fig. 2 One-dimensional spatial spectra measured simultaneously by wave-staff array (SRI) and optical (RRI) technique.

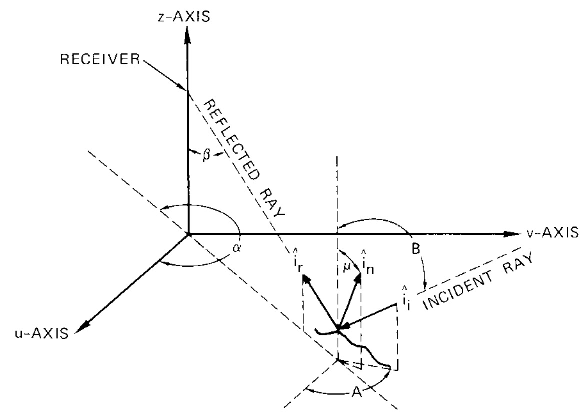

Fig. 3 Geometry for optical reflection techniques.

light in terms of the angular location of the source (A,B) and the angular location of the receiver (α, β).

Using this condition, the ray-optics analysis allows one to write the slope PDF, $p(W_u, W_v)$ in terms of known geometric factors (α, β, A, B), the known surface irradiance H from the light source, the known Fresnel reflectance, ρ, and the experimentally determined average surface radiance $N(\alpha, \beta, A, B)$. That is,

$$p(W_u, W_v) = \left[\frac{\cos\beta}{\rho H} \right] N(\alpha, \beta, A, B) \times \left[\frac{(\cos\beta + \cos B)^2}{1 + \cos\beta \cos B + \sin\beta \sin B \cos(\alpha - A)} \right]^2 \quad (1)$$

where W_u and W_v are the u-component and v-component of slope respectively.

In the case of the Cox and Munk[7] experiments (see Fig. 4) the surface radiance was averaged by convolving the point spread function of a pinhole camera with the actual surface-radiance function. This procedure effectively averaged the surface radiance over an area of about 4600 m^2, from which one would expect a reasonable ensemble average with one exposure of the film.

Certain advantages accrue to this technique when an active source such as a laser is used rather than the sun or moon. The principal advantage is better background discrimination, and is derived from the fact than an active source can be modulated and the fact that, with a laser, a narrowband filter can be used to discriminate against a spectrally broad background. While adding a laser to the system increases the cost and complexity of the experiment, the attendant advantage is very significant in view of the fact that, with sun illumination, the technique is limited at large slopes by background from reflected sky light and upwelling radiation from the ocean. It should be mentioned that the temporal coherence of a laser could be a disadvantage rather than an advantage if the coherence length were too long. However, temporal coherence is not a problem when suitable geometry is used and/or a laser with coherence length much shorter than the rms surface height is used. Finally, an active source has the obvious advantage of permitting freedom in the choice of the experimental geometry and the time of day for the experiment.

2. Curvature-Probability Sensors

The measurement of curvature PDF has been confined to laboratory wave tanks rather than real ocean waters, although

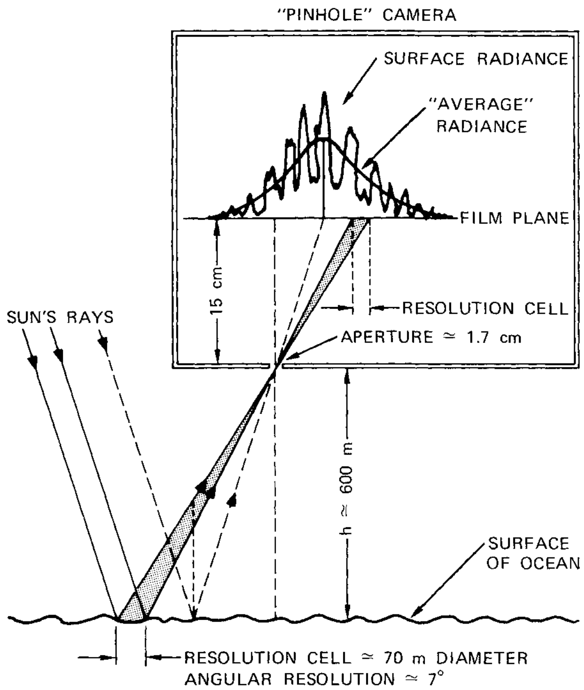

Fig. 4 Cox and Munk technique for measurement of slope probability density function.

there is no fundamental reason for this restriction. Both the Schooley[10] technique and the Wu[11] technique (see Fig. 5) infer curvature at a known slope by measuring the size of specular "glints" at the image plane of a lens.

In the Schooley experiments a small circular diffuse light source illuminates the water, and the resulting specular "glints" are recorded on film by a camera focused on the surface of the water. The slope at which the curvature is measured is determined by the geometry of the apparatus and is indicated by the location of a specular "glint" on the film. Although the technique basically records two-dimensional curvature information, Schooley analyzed only the data for curvature along the direction of the wind. He concluded from rather limited data that the probability for a given curvature was essentially independent of slope.

In the Wu experiments a linear diffuse light source, small in one dimension and effectively infinite in the other dimension, illuminates the water, and the flux through a small circular aperture at the image plane of a lens is recorded as a function of time. The slope at which the curvature is measured is determined by the geometry of the apparatus. This is basically a one-dimensional technique and gives the probability for curvature along the wind direction only. Wu demonstrated with his data on cylindrical waves that the curvature-probability distribution was not independent of slope.

Both techniques are based on the assumption that the surface surrounding a point can be treated as a convex or concave mirror whose topography can be described by two principal radii of curvature. Under this assumption and the restriction that the surface-to-camera distance is large compared to both the rms surface height and the radii of curvature, it is possible to show that the size of a "glint" in a given direction is inversely proportional to the curvature in that direction. In this way, curvature can be inferred from a measure of the "glint" size. Although both techniques fail when the curvatures become small, they do provide probability distributions over a useful range of curvatures and could, in principle, be extended to provide two-dimensional probability distributions in real ocean waters.

3. *Spatial PSD Sensors*

The earliest attempt to measure the wavenumber spectrum of the ocean was undertaken by the Stereo Wave Observation

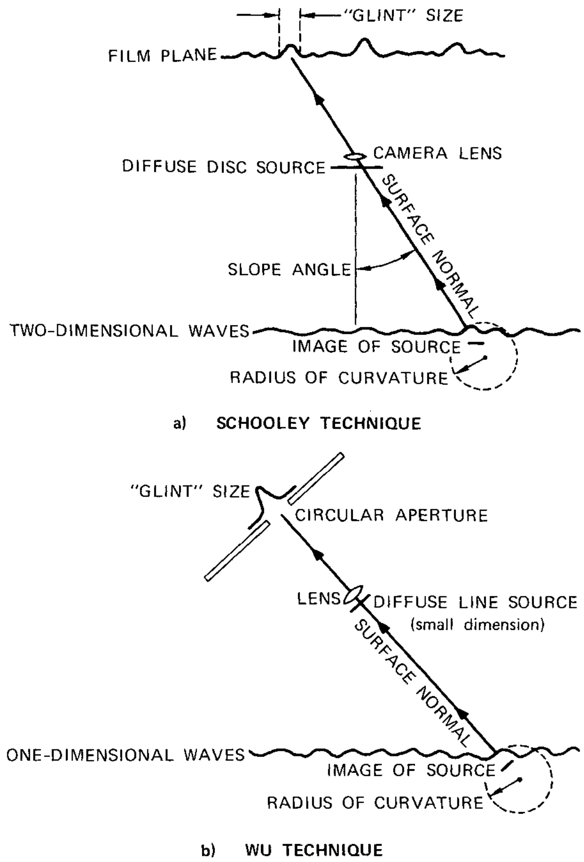

Fig. 5 Techniques for measurement of curvature probability density function.

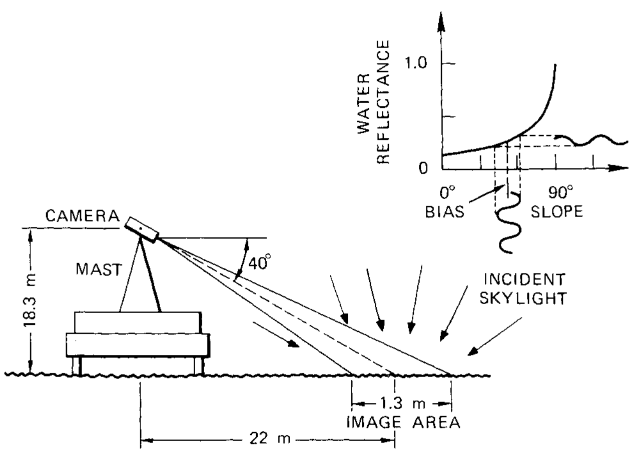

Fig. 6 Stilwell technique for measurement of spatial slope PSD function.

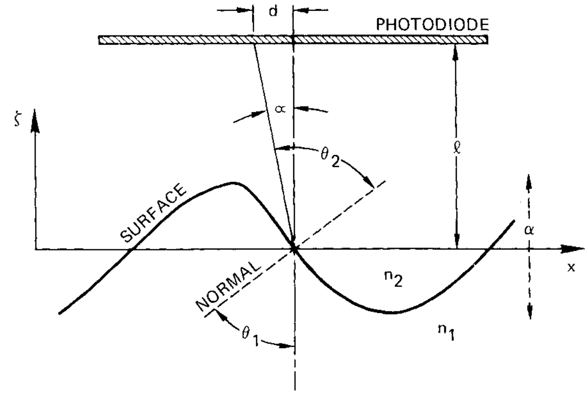

Fig. 7 Optical path and refraction angles for measurement of temporal slope fluctuations.

Project.[12] In this technique, photographs of the sea were taken using omnidirectional sky illumination. In the SWOP project, the photographs were taken from two airplanes flying in a precise pattern; however, Dobson used a ship[13] as a platform. From the resulting stereo-photographs, the wave heights were determined as a function of position. The data were corrected for various sources of noise and measurement error, and the spatial correlation coefficient of the surface was determined. The wavenumber spectrum was then obtained. Aside from the difficulties of obtaining the stereo pair, the data reduction involved in this technique is tedious and slow. Moreover, the method is limited to daylight operation.

Stilwell[14] suggested a procedure in which a photograph of the ocean surface was recorded using omnidirectional illumination. The reflectance of water is a function of surface slope; thus the surface radiance from each point on the surface is modulated as the local slope changes (Fig. 6). If the surface illumination is uniform, the two-dimensional slope distribution on the ocean surface can be related to the two-dimensional intensity distribution in the image plane of a camera. The Fourier transform of the two-dimensional intensity distribution, $F\{I\}$, in the image plane of the lens is given by

$$|F\{I\}|^2 = \left[\frac{\partial I}{\partial \omega} \right]^2 k^2 S(k, \phi) \cos^2(\alpha - \phi) \qquad (2)$$

where I is the intensity of light reflected from a point on the ocean surface, ω is the angle of reflection, $(\alpha-\phi)$ is the angle between the direction of look and the direction of the wave vector k and $S(k, \phi)$ is the spatial PSD of the wave-height fluctuations.

The wave-slope PSD is the product of wavenumber squared and the wave-height PSD. For this equation to be valid, it is essential that the sky irradiation be uniform, that the slopes range over small angles (so that the reflectance is nearly linearly dependent on slope), and that the declination angle be small. Evidently, the operation will be limited to daylight hours.

Stilwell originally used film recording and an optical method to perform the Fourier transform and obtain the two-dimensional spatial PSD. King and co-workers replaced the film recording with a TV camera and used a digital method to obtain the Fourier Transform, thereby removing some of the troublesome and time-consuming steps. At present, however, only a one-dimensional Fourier transform is made in real time.

A comparison of one-dimensional spatial spectra of wave-height fluctuations obtained by this technique (as measured by King and Lizzi of Riverside Research Institute) and from wave-staff arrays (measured by SRI) is shown in Fig. 2. The measurements were made at the Naval Undersea Center tower in San Diego. The data for the optical technique are represented by the dots. The line represents the spectral estimate inferred from the wave-staff array. The agreement between the two spectral estimates is excellent.

C. Optical Refraction

If an optical beam is incident on the surface from below, the transmitted beam will be refracted according to Snell's law. The angular deviation of the transmitted beam from the vertical is directly related to the local slope. In 1958, Cox[15] originally used such a technique. Recently, Tober and co-workers[16] and Sturm and co-workers[17] (see Fig. 7) have modified the techniques using a laser source. The technique of Tober et al.[16] in particular is capable of measuring two-dimensional slopes. These experimenters have measured waves of amplitudes up to 5 cm and slopes up to 20°, with a short-wavelength limit of 1.7 mm. In addition to its use in laboratory wind-wave tanks, the technique has been used in the ocean from a platform attached to the bow of a ship.[18]

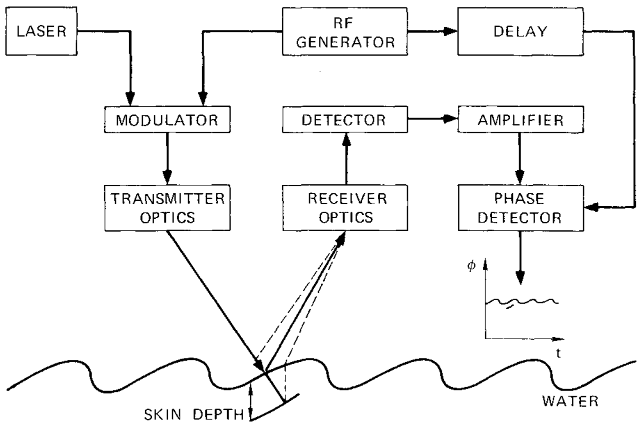

Fig. 8 Block diagram of profilometer wave-height-measurement system.

The requirement of intercepting the transmitted ray restricts the methods to small slopes for moderate wave amplitudes. The technique is not a means for remote measurement of the surface slope.

D. Laser Profilometers

Laser profilometers,[19,20] used to measure wave heights, are based on phase-ranging with amplitude-modulated laser radiation. A schematic of such a system is shown in Fig. 8. The backscattered radiation consists of two components – the specular reflection from appropriately oriented facets, and the radiation diffusely scattered by the bulk of the water and/or surface imperfections. Of the two, the former is higher in magnitude but the illuminated area must be large to assure the presence of at least one properly oriented facet within that area. If the optical beam is so small that the probability that a glint does not occur within it is high, the backscattered radiation is from the depth corresponding to the skin depth of the radiation. At 6328 Å, the skin depth for clear water is of the order of meters. As a consequence, the

Table 1 Summary of surface-wave sensors

Technique	Measurement	Comments
Wave-staff arrays	Spatial PSD Temporal PSD PDF Wave direction Dispersion relation	Wavelength resolution to 1 cm Contacting
Stereo photometry	Spatial PSD PDF	Data processing tedious Daylight hours, uniform sky radiance, remote Wave-length resolution to 1 cm
Stilwell/RRI	Spatial scope PSD Slope PDF	Remote, near proximity Daylight hours, uniform sky radiance
Profilometer	Temporal PSD PDF	Remote Wavelength resolution to tens of cms
Specular glints	Slope PDF Curvature PDF	Remote
Radar	Ampl. fluctuations of resonant wavelength	Remote Narrow wavelength band response
Optical refraction	Temporal scope PSD Slope PDSF	Short-wavelength resolution to 1.7 mm Contacting

beam diameter must be kept large (with its attendant poor lateral resolution) to guarantee specular surface glints. A He-Ne laser profilometer based on the specular reflection has been used to measure wave heights to an accuracy of a few centimeters with a lateral resolution of tens of centimeters at a range of 60 m by Schule and co-workers.[19] For infrared lasers, the skin depth is considerably less than for visible lasers, typically a few tens of microns. Thus a profilometer based on the diffuse backscatter from a CO_2 laser can measure wave heights to within a few millimeters with a lateral resolution of a few millimeters at ranges of upto 10 m. A single profilometer can be used to develop the temporal power spectrum. Several profilometers or a scanning profilometer can produce the spatial power spectrum.

E. Radar Systems

Microwave (acoustic) radars measuring sea-surface scatter are Bragg scattering systems—i.e., they see the amplitude fluctuations of the surface wavelength that is resonant to the electromagnetic (acoustic) wavelength. For example, if the radar system transmits a wavelength λ, and the angle between the incident beam and the normal to the surface is Θ, the radar system tracks the amplitude fluctuations of surface waves of wavelength ($\lambda/2 \cos\theta$). Of course, this technique is very narrowband; nevertheless it is a means of continuously monitoring the amplitude and phase velocity fluctuations (with a coherent radar) of very-short-wavelength components remotely on a 24-hour basis.

III. Summary

This paper has discussed the variety of surface-wave-sensing techniques. They are summarized in Table 1. This table is an indication of the current state-of-the-art, and does not preclude improvements. The choice of a technique, for any situation, depends on the nature of the information desired and the experimental constraints.

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